SIGNAL CLASSIFICATION THROUGH MULTIFRACTAL ANALYSIS AND COMPLEX DOMAIN NEURAL NETWORKS

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Abstract

This paper describes a system capable of classifying stochastic, self-affine, nonstationary signals produced by nonlinear systems. The classification and analysis of these signals is important because they are generated by many real-world processes. The first stage of the signal classification process entails the transformation of the signal into the multifractal dimension domain, through the computation of the variance fractal dimension trajectory (VFDT). Features can then be extracted from the VFDT using a Kohonen self-organizing feature map. The second stage involves the use of a complex domain neural network and a probabilistic neural network to determine the class of a signal based on these extracted features. The results of this paper show that these techniques can be successful in creating a classification system which can obtain correct classification rates of about 87% when performing classification of such signals with an unknown number of classes.

Keywords: *Multifractal analysis; complex domain neural networks; probabilistic neural networks; classification*

1. INTRODUCTION

This paper investigates the development of a software system that is capable of classifying stochastic, self-affine, nonstationary signals that originate from nonlinear systems. Such signals are often multivariate, and the system described in this paper will have the ability to take these multivariate signals into account during the classification process.

The features used for classification are based on a temporal multifractal characterization of the signal, which is achieved through the computation of its variance fractal dimension trajectory (VFDT) [Kins94]. This translation into the temporal multifractal dimension domain emphasizes the underlying complexity of the signal, and more importantly for classification, has a normalizing effect. The classification based on these features is performed by a complex domain neural network that can operate upon signal features from separate, but strongly correlated signals without losing the correlation between the signals. Furthermore, complex domain neural networks often generalize more effectively and train faster than their real-valued counterparts.

While the classification system implemented for this paper is not specific to any particular signal, spatio-temporal recordings of a Siamese fighting fish when presented with various stimuli during dishabituation experiments were used to evaluate the performance of the system. A stereoscopic camera system was used to track and record the three dimensional Cartesian co-ordinates of the fish over an eight hour period. A sample of this dishabituation signal is shown in Fig. 1. Stimuli applied during these experiments were on the Y-Z plane at $X = 0$ and on the X-Y plane at approximately $Z = 225$. Since there were no stimuli along the Y-axis and it was the least accurate because it was resolved indirectly through the stereoscopic vision, the Y-component of the signal was not used for classification in this paper [ChCa03]. An added difficulty in analyzing these signals was that they contained an unknown number of classes, but this was overcome using clustering algorithms.

An overview of the techniques used in this paper for classification is provided in Sec. 2. Details of the experiments performed with the dishabituation signals and the classification system are presented in Sec. 3.

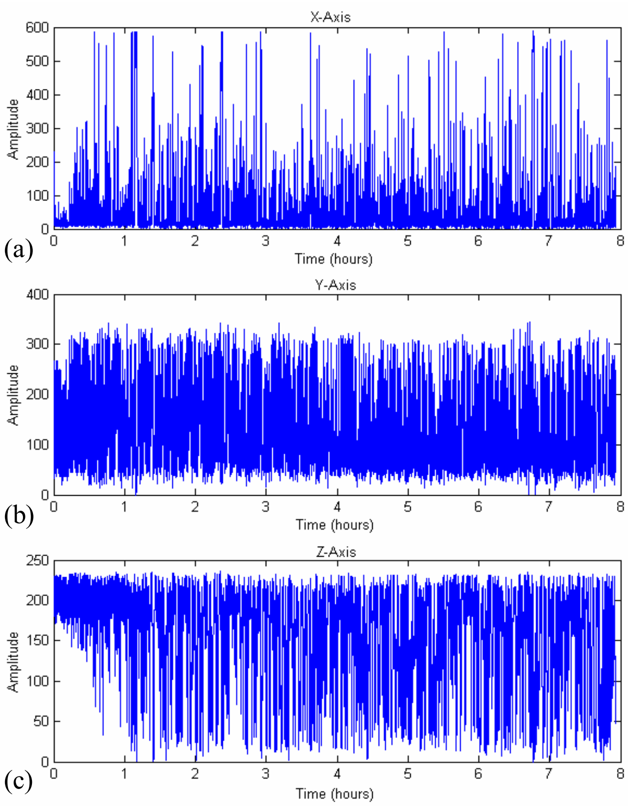

Fig. 1. Fish trajectory signal along the X (a), Y (b), and Z(c) directions.

2. BACKGROUND

2.1 Variance Fractal Dimension Trajectory

The feature extraction technique considered for this paper is a translation into the temporal multifractal dimension domain by computing a VFDT [Kins94]. An advantage of using the variance fractal dimension trajectory for classification is that it emphasizes the underlying complexity of the signal, thus helping to provide the unique identification for each class. Another advantage is that the transformation provides a normalizing effect because the theoretical range of fractional dimensionality of Euclidean one-dimensional signal is between 1 and 2.

The variance fractal dimension is based on calculations involving the variance of the amplitude increments of a signal taken at different scales. The amplitude increments of a signal, *x(t)*, over a time interval *Δt* follow the following power law relationship

$$Var[x(t_2) - x(t_1)] \sim |t_2 - t_1|^{2H} \qquad (1)$$

where H is the Hurst exponent. The Hurst exponent can be calculated via a log-log plot using

$$H = \lim_{\Delta t \to 0} \frac{1}{2} \frac{\log[Var(\Delta x)_{\Delta t}]}{\log(\Delta t)} \qquad (2)$$

The variance fractal dimension, D_σ, is then given by

$$D_\sigma = E + 1 - H \qquad (3)$$

where E is the Euclidean dimension.

The process of calculating the VFDT of a signal involves computing the variance fractal dimension in a sliding window fashion. The selection of the window size is based on the stationarity of the signal. This representation of a signal by its VFDT acts as the features used to perform classification.

2.2 Complex Domain Neural Network

Complex domain three-layer feedforward neural networks are used in this paper to perform classification based upon the variance fractal dimensions.

Neural networks that work with real-valued inputs are sufficient for most situations, but when the inputs to the neural network are naturally represented as complex numbers, it is advantageous to use a neural network that takes this representation into account. Complex valued data can be provided to a real domain neural network by separating the components of the complex values and providing them separately as inputs; however, the strong correlation between the components is lost. While in theory, real valued neural networks have the same ability as complex domain neural networks, in practice, the training of complex domain neural networks is typically faster and they often generalize better, especially when only a sparse training set is available.

The architecture of complex domain three-layer feedforward neural networks are similar to their real domain counterparts; the main differences are that each input value and weight is a complex number consisting of both a real and imaginary part. The activation function used in this paper for the neurons is a scaled version of the hyperbolic tangent function, tanh(1.5x), which is applied to the magnitude of the complex valued input and then multiplied by the unit vector of the input so that the output of the activation function maintains the same direction as the input [Mast94].

This paper uses a single output neuron for each class in order to perform classification. Since the output neurons result in binary decisions for the inclusion or exclusion of an input to a particular class, it is inefficient to employ complex-valued outputs as it does not aid in making the classification decision. Thus, for classification purposes, the imaginary part of the output of these neurons is discarded and the decisions are based solely upon the real part of the output.

The training of the network is performed using the standard backpropagation algorithm extended to operate with complex values. The partial derivates of the error of the output with respect to the real and imaginary parts of the weights is used as the error gradient to indicate the direction with which to modify the weights. The modifications to the weights in each epoch is given by

$$w_{new_{real}} = w_{old_{real}} - \alpha \frac{\partial \varepsilon}{\partial w_{old_{real}}} \tag{4a}$$

$$w_{new_{imag}} = w_{old_{imag}} - \alpha \frac{\partial \varepsilon}{\partial w_{old_{imag}}} \tag{4b}$$

where ε is the output error, w is the weight in the network, the *real* and *imag* subscripts indicate the real and imaginary parts of the weights, and α is the learning rate.

2.3 Probabilistic Neural Network

An alternative to the complex domain neural network for classification is the probabilistic neural network. The probabilistic neural network (PNN) is an implementation of the Bayes optimal decision rule in the form of a neural network [Spec88]. PNNs have a number of advantages over traditional neural networks in that they tend to train orders of magnitude faster and their classification accuracy asymptotically approaches Bayes optimal.

2.4 Kohonen Self-Organizing Feature Map

Kohonen self-organizing feature maps (SOFMs) [Koho84] can be used as a clustering algorithm to determine the classes of signals. SOFMs can also be used to perform feature extraction upon the VFDT prior to classification. SOFMs are neural networks that employ unsupervised competitive learning algorithms. These neural networks are referred to as topology-preserving in that the neighbourhood relations of the data are preserved and structure is imposed upon the neurons in the network. This clustering of the data based on their relations allows for the discovery of the underlying structure of the data.

3. EXPERIMENTAL WORK

3.1 Experiment Setup

The dishabituation signals were segmented into lengths of 4096 samples and the classes of each of the segments were determined through clustering of the X and Z-axis segments in the time domain with a Kohonen self-organizing feature map [ChCa03]. To train the classification system, a training set of 612 segments from 9 recordings was used. The testing set applied to the system was made up of 544 segments from 8 recordings that were not used for the training set.

No filtering was performed upon the signals prior to the computation of the VFDTs used to construct the training and testing sets. The VFDTs were computed using a window size of 2048 samples, the largest window in which the fractal dimension of the signals remained constant. A window displacement of 256 samples was used, as it was discovered to give a good resolution of the VFDT.

Figure 2b shows the VFDT of the signal of Fig. 2a. The first thing to note about the VFDT plot is that the fractal dimensions of the signal changes, indicating that it is multifractal in time. It can further be noted that the samples of the VFDT are normalized dimensions between 1 and 2, which is essential for the classification process. Additionally, the VFDT plots visually seem to correspond to the time domain plots in that they tend to emphasize some of the characteristics in the original signal; the most exemplary characteristic being the initial large changes in the VFDT signals which correspond to the irregular motion of the fish as seen in the time domain plot.

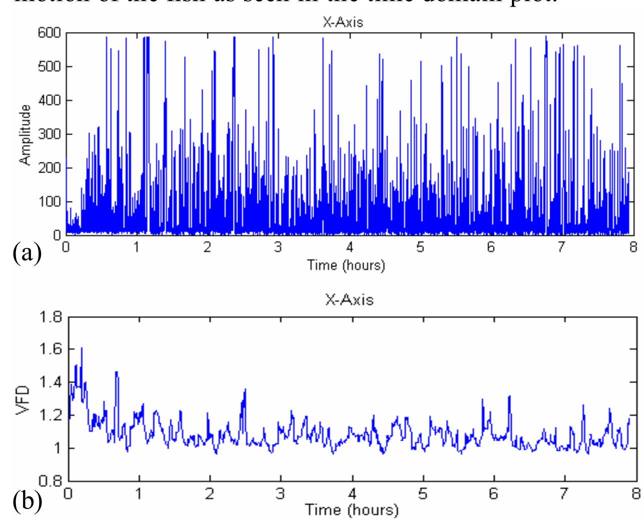

Fig. 2. Fish trajectory signal (a) and its VFDT (b).

3.2 CNN Experiment

The results of the classification of the input vectors in the testing set using the complex domain neural network (CNN) are shown in the confusion matrix in Table 1. Overall, the classification system performed well at a correct classification rate of nearly 87%.

The size of each class in the training and testing sets were proportional to their frequency of occurrence in the signals. While the first class had the smallest representation, it was so distinct that all but one of input

Table 1. CNN experiment confusion matrix.

		Experimental 1	Experimental 2	Experimental 3	Experimental 4	Correct Classification Rate (%)
Expected	1	23	0	0	1	95.83
	2	3	127	8	8	86.99
	3	0	11	151	26	80.32
	4	0	13	3	170	91.40
Average Correct Classification Rate: 86.58%						
95% Confidence Interval: [83.72%, 89.44%]						

vectors of this class were correctly classified. Input vectors from the remaining classes were also classified at a high rate, giving confidence to the abilities of the system. As the development of the testing set involved randomness in selecting the input vectors to use for testing, the 95% confidence interval for the classification rate is provided under the confusion matrix in order to bound the true classification rate of the system.

3.3 Additional Experiments

Additional experiments were performed using a PNN as the classifier and the results are shown in Table 2. For the first experiment, the X-axis fractal dimensions were used for classification by the PNN. The second experiment was identical to the first, except that the Z-axis signals were used. In both experiments, the results were quite poor. However, by utilizing both the X and Z-axis fractal dimensions for classification, a significantly higher classification rate was achieved.

Table 2. PNN experiments.

Signal	Classification Rate (%) 1	2	3	4	Average Classification Rate (%)
X	100	92	59	50	67
Z	63	29	47	91	58
X & Z	100	95	84	95	91

While the results for this last experiment gave slightly higher classification rates than those with the CNN, they are comparable when confidence intervals are taken into account. However, there were some differences in the training and execution times. The PNN trained two orders of magnitude faster than the CNN, while the trained CNN performed classification nearly an order of magnitude faster than the PNN.

These experiments were also repeated using SOFMs to perform feature extraction upon the VFDT prior to classification. For most cases, the classification results when using the SOFMs were slightly lower than when the SOFMs were excluded, but they are essentially equal when confidence intervals are taken into account. Thus, the classification rates remained almost the same despite the fact that fewer features were used for classification.

Further details of these additional experiments can be found in [ChCa03].

4. CONCLUSIONS

This work was done to demonstrate the feasibility of classification of self-affine signals by using variance fractal dimensions and complex domain neural networks. This paper has shown that a multifractal characterization of self-affine signals through variance fractal dimensions is an effective means of feature extraction as it provided a sufficient metric upon which to classify the signals used in this paper. Furthermore, the use of complex domain neural networks upon two separate, yet strongly correlated signals were used and shown to be effective in classifying these signals based on its variance fractal dimensions.

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